

PULITZER PRIZE WINNER

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GÖDEL, ESCHER, BACH: an Eternal Golden Braid

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A metaphysical inquiry on mind, art, and machines in the work of Kurt Gödel

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Table of Contents

- [Part II: EGB](#)
- [Introduction:](#)
- [A Musico-Logical Offering](#)
- [Three-Part Invention](#)
- [Formal Systems](#)
- [Theorems, Axioms, Rules](#)
- [Inside and Outside the System](#)
- [Jumping out of the System](#)
- [M-Mode, I-Mode, U-Mode](#)
- [Decision Procedures](#)
- [or,](#)
- [What the Tortoise Said to Achilles](#)
- [by Lewis Carroll'](#)
- [The pq-System](#)
- [The Decision Procedure](#)
- [Bottom-up vs. Top-down](#)
- [Isomorphisms Induce Meaning](#)
- [Meaningless and Meaningful Interpretations](#)
- [Active vs. Passive Meanings](#)
- [Double-Entendre!](#)
- [Formal Systems and Reality](#)
- [Mathematics and Symbol Manipulation](#)
- [The Basic Laws of Arithmetic](#)
- [////////](#)
- [Ideal Numbers](#)
- [Euclid's Proof](#)
- [Getting Around Infinity](#)
- [The telephone rings; Achilles picks it up.](#)
- [Primes vs. Composites](#)
- [The tq-System](#)
- [Capturing Compositeness](#)
- [Illegally Characterizing Primes](#)
- [Figure and Ground in Music](#)

- ["Bottoming Out" and Heterarchies](#)
- [Expanding Nodes](#)
- [Diagram G and Recursive Sequences](#)
- [A Chaotic Sequence](#)
- [Two Striking Recursive Graphs](#)
- [Recursion at the Lowest Level of Matter](#)
- [Copies and Sameness](#)
- [Programming and Recursion: Modularity, Loops, Procedures](#)
- [Recursion in Chess Programs](#)
- [Recursion and Unpredictability](#)
- [When Is One Thing Not Always the Same?](#)
- [Information-Bearers and Information- Revealers](#)
- [Genotype and Phenotype](#)
- [Exotic and Prosaic Isomorphisms](#)
- [Jukeboxes and Triggers](#)
- [DNA and the Necessity of Chemical Context](#)
- [An Unlikely UFO](#)
- [Levels of Understanding of a Message](#)
- ["Imaginary Spacescape"](#)
- [The Heroic Decipherers](#)
- [Three Layers of Any Message](#)
- [Schrodinger's Aperiodic Crystals](#)
- [Languages for the Three Levels](#)
- [The "Jukebox" Theory of Meaning.](#)
- [Against the Jukebox Theory](#)
- [Meaning Is Intrinsic If Intelligence Is Natural](#)
- [Earth Chauvinism](#)
- [Two Plaques in Space](#)
- [Bach vs. Cage Again](#)
- [How Universal Is DNA's Message?](#)
- [Words and Symbols](#)
- [Alphabet and First Rule of the Propositional Calculus](#)
- [Well-Formed Strings](#)
- [More Rules of Inference](#)
- [The Fantasy Rule](#)
- [Recursion and the Fantasy Rule](#)

- [The Converse of the Fantasy Rule](#)
- [The Intended Interpretation of the Symbols](#)
- [Rounding Out the List of Rules](#)
- [Justifying the Rules](#)
- [Semi-Interpretations](#)
- [Is There a Decision Procedure for Theorems?](#)
- [Do We Know the System Is Consistent?](#)
- [The Carroll Dialogue Again](#)
- [Shortcuts and Derived Rules](#)
- [Formalizing Higher Levels](#)
- [Reflections on the Strengths and Weaknesses of the System](#)
- [Proofs vs. Derivations](#)
- [The Handling of Contradictions](#)
- [The Crab Canon and Indirect Self-Reference](#)
- [What We Want to Be Able to Express in TNT](#)
- [Numerals](#)
- [Variables and Terms](#)
- [Atoms and Propositional Symbols](#)
- [Free Variables and Quantifiers](#)
- [Translating Our Sample Sentences](#)
- [Translation Puzzles for You](#)
- [How to Distinguish True from False?](#)
- [The Rules of Well-Formedness](#)
- [A Few More Translation Exercises](#)
- [A Non typographical System](#)
- [The Five Axioms and First Rules of TNT](#)
- [The Five Peano Postulates](#)
- [New Rules of TNT: Specification and Generalization](#)
- [The Existential Quantifier](#)
- [Rules of Equality and Successorship](#)
- [Illegal Shortcuts](#)
- [Why Specification and Generalization Are Restricted](#)
- [Something Is Missing](#)
- [-Incomplete Systems and Undecidable Strings](#)
- [Non-Euclidean TNT](#)
- [The Last Rule](#)

- [Tension and Resolution in TNT](#)
- [Formal Reasoning vs. Informal Reasoning](#)
- [Number Theorists Go out of Business](#)
- [Hilbert's Program](#)
- [What Is Zen?](#)
- [Zen Master Mumon](#)
- [Zen's Struggle Against Dualism](#)
- [Ism, The Un-Mode, and Unmon](#)
- [Zen and Tumbolia](#)
- [Escher and Zen](#)
- [Hemiolia and Escher](#)
- [Indra's Net](#)
- [Mumon on MU](#)
- [From Mumon to the MU-puzzle](#)
- [Mumon Shows Us How to Solve the MU-puzzle](#)
- [Gödel-Numbering the MIU-System](#)
- [Seeing Things Both Typographically and Arithmetically](#)
- [MIU-Produced Numbers](#)
- [Answering Questions about Produced Numbers by Consulting TNT](#)
- [The Dual Nature of MUMON](#)
- [Codes and Implicit Meaning](#)
- [The Boomerang: Gödel-Numbering TNT](#)
- [TNT-Numbers: A Recursively Enumerable Set of Numbers](#)
- [Mumon Has the Last Word](#)
- [À](#)
- [Levels of Description](#)
- [Chunking and Chess Skill](#)
- [Similar Levels](#)
- [Computer Systems](#)
- [Instructions and Data](#)
- [Superconductivity: A "Paradox" of Renormalization](#)
- ["Sealing-off"](#)
- [... then, one by one, the four voices of the fugue chime in.\)](#)
- [New Perspectives on Thought](#)
- [Intensionality and Extensionality](#)
- [The Brain's "Ants"](#)

- [Larger Structures in the Brain](#)
- [Mappings between Brains](#)
- [A "Grandmother Cell"?](#)
- [Classes and Instances](#)
- [The Prototype Principle](#)
- [Symbols -Software or Hardware?](#)
- [Liftability of Intelligence](#)
- [Can One Symbol Be Isolated?](#)
- [The Symbols of Insects](#)
- [Class Symbols and Imaginary Worlds](#)
- [Intuitive Laws of Physics](#)
- [Procedural and Declarative Knowledge](#)
- [Visual Imagery](#)
- [Minds and Thoughts](#)
- [CHAPTER XI11](#)
- [Self-Awareness and Chaos](#)
- [Representability and Refrigerators](#)
- [Ganto's Ax in Metamathematics](#)
- [Finding Order by Choosing the Right Filter](#)
- [Primordial Steps of the Language BlooP](#)
- [Loops and Upper Bounds](#)
- [Conventions of BlooP](#)
- [IF-Statements and Branching](#)
- [Automatic Chunking](#)
- [BlooP Tests](#)
- [BlooP Programs Contain Chains of Procedures](#)
- [Suggested Exercises](#)
- [Primitive Recursive Predicates Are Represented in TNT](#)
- [Are There Functions Which Are Not Primitive Recursive?](#)
- [Pool B, Index Numbers, and Blue Programs](#)
- [The Diagonal Method](#)
- [Cantor's Original Diagonal Argument](#)
- [What Does a Diagonal Argument Prove?](#)
- [The Insidious Repeatability of the Diagonal Argument](#)
- [From BlooP to FlooP](#)
- [Terminating and Nonterminating FlooP Programs](#)

- [Turing's Trickery](#)
- [A Termination Tester Would Be Magical](#)
- [Pool F, Index Numbers, and Green Programs](#)
- [The Termination Tester Gives Us Red Programs](#)
- [Gloop ...](#)
- [... Is a Myth](#)
- [The Church-Turing Thesis](#)
- [Terminology: General and Partial Recursive](#)
- [The Power of TNT](#)
- [Air on G's String](#)
- [CHAPTER XIV](#)
- [The Two Ideas of the "Oyster"](#)
- [The First Idea: Proof-Pairs](#)
- [Proof-Pair-ness Is Primitive Recursive...](#)
- [... And Is Therefore Represented in TNT](#)
- [The Power of Proof-Pairs](#)
- [Substitution Leads to the Second Idea](#)
- [The Last Straw](#)
- [TNT Is \$\epsilon\$ -Incomplete](#)
- [Multifurcation](#)
- [The Passion According to Lucas](#)
- [Jumping Up a Dimension](#)
- [The Limits of Intelligent Systems](#)
- [There Is No Recursive Rule for Naming Ordinals.](#)
- [Self-Transcendence-A Modern Myth](#)
- [Advertisement and Framing Devices](#)
- [Simplicio, Salviati, Sagredo: Why Three?](#)
- [Zen and "Stepping Out"](#)
- [CHAPTER XVII](#)
- [Formal and Informal Systems](#)
- [Intuition and the Magnificent Crab](#)
- [The Church-Turing Thesis](#)
- [The Public-Processes Version](#)
- [Srinivasa Ramanujan](#)
- ["Idiots 'Savants"](#)
- [The Isomorphism Version of the Church-Turing Thesis](#)

- [Representation of Knowledge about the Real World](#)
- [Processes That Are Not So Skimmable](#)
- [Articles of Reductionistic Faith](#)
- [Parallel Progress in AI and Brain Simulation?](#)
- [Beauty, the Crab, and the Soul](#)
- [Irrational and Rational Can Coexist on Different Levels](#)
- [More Against Lucas](#)
- [An Underpinning of AI](#)
- [Church's Theorem](#)
- [Tarski's Theorem](#)
- [The Impossibility of the Magnificrab](#)
- [Two Types of Form](#)
- [Beauty, Truth, and Form](#)
- [The Neural Substrate of the Epimenides Paradox](#)
- [Turing](#)
- [The Turing Test](#)
- [Turing Anticipates Objections](#)
- [Samuel's Checker Program](#)
- [When Is a Program Original?](#)
- [Theorem Proving and Problem Reduction](#)
- [Shandy and the Bone](#)
- [Changing the Problem Space](#)
- [Applying AI to Mathematics](#)
- [The Crux of AI: Representation of Knowledge](#)
- [DNA and Proteins Help Give Some Perspective](#)
- [Modularity of Knowledge](#)
- [Representing Knowledge in a Logical Formalism](#)
- [Deductive vs. Analogical Awareness](#)
- [From Computer Haiku to an RTN-Grammar](#)
- [From RTN's to ATN's](#)
- [A Little Turing Test](#)
- [Images of What Thought Is](#)
- [Higher-Level Grammars ...](#)
- [Grammars for Music?](#)
- [Winograd's Program SHRDLU](#)
- [The Structure of SHRDLU](#)

- [PLANNER Facilitates Problem Reduction](#)
- [Syntax and Semantics](#)
- ["Almost" Situations and Subjunctives](#)
- [Layers of Stability](#)
- [Frames and Nested Contexts](#)
- [Preprocessing Selects a Mini-vocabulary](#)
- [High-Level Descriptions](#)
- [Templates and Sameness-Detectors](#)
- [A Heterarchical Program](#)
- [The Concept Network](#)
- [Slippage and Tentativity](#)
- [Meta- Descriptions](#)
- [Flexibility Is Important](#)
- [Focusing and Filtering](#)
- [Science and the World of Bongard Problems](#)
- [Connections to Other Types of Thought](#)
- [Message-Passing Languages, Frames, and Symbols](#)
- [Enzymes and AI](#)
- [Fission and Fusion](#)
- [Epigenesis of the Crab Canon](#)
- [Conceptual Skeletons and Conceptual Mapping](#)
- [Recombinant Ideas](#)
- [Abstractions, Skeletons, Analogies](#)
- [Multiple Representations](#)
- [Ports of Access](#)
- [Forced Matching](#)
- [Recap](#)
- [Creativity and Randomness](#)
- [Picking up Patterns on All Levels](#)
- [The Flexibility of Language](#)
- [Intelligence and Emotions](#)
- [AI Has Far to Go](#)
- [Ten Questions and Speculations](#)
- [Can Machines Possess Originality?](#)
- [Below Every Tangled Hierarchy Lies An Inviolable Level](#)
- [A Self-Modifying Game](#)